A Game-Theoretic Analysis of Pricing Competition between Aggregators in V2G Systems

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***Abstract*—While the Plug-in Electric Vehicles (PEVs) are gaining popularity, Vehicle-to-Grid (V2G) technology is becoming a reality. In V2G, a PEV provides energy as well as consumes it. Since the battery of a PEV can store a small amount of electric power, a large number of PEVs must be combined to offer useful services to the grid. However, these vehicles must be managed to provide controlled services according to the need of the grid leading to the introduction of an aggregator. This work assumes a system of multiple aggregators to which a PEV can choose to subscribe. An aggregator charges its subscribers for the V2G services. To maximize their profits, the aggregators vie with each other to get the market share. This competition dictates the prices and it is crucial for each competitor to choose an optimal price. The competition among aggregators is influenced by several factors. In this work, we analyze this competition by modeling the problem as a sequential game, in particular, using the Stackelberg Leadership Model. Solving the model provides the optimal prices. We also analyze the same problem by modeling it as a simultaneous game using the Cournot Competition Model and compare the game results with that of the Stackelberg game. We conduct an extensive evaluation of the game results to demonstrate the influence of different factors on optimal behavior.**

***Index Terms*—Vehicle-to-Grid; Plug-in Electric Vehicles; Game Theory; Optimal Pricing.**

I. Introduction

Because they are environmentally friendly and economically efficient, PEVs are expected to become the transportation mode of choice in most countries by the coming decade. The sale of electric vehicles is expected to increase to 41 million by 2040, which represents 35% of new light-duty vehicle sales [1]. The growing number of PEVs takes us close to the real establishment of the Vehicle-to-Grid (V2G) technology. The PEVs, while plugged-in, can be used as a small and distributed storage mechanism by the grid. Although the battery of a PEV can store some kW electric power, a larger number of PEVs can provide a combined storage which can be useful as an energy source or an ancillary (*e.g.*, frequency regulation) service provider. Hence, the notion of an aggregator is introduced for the V2G management. PEVs subscribe to an aggregator according to their availability, while the aggregator manages the V2G services [2]. A PEV subscribes to an aggregator and pays the aggregator when the PEV's battery is charged. The aggregator, on the other hand, buys the same electric power from the grid. When the PEV sells (discharges) electricity to the aggregator, it gets paid by the aggregator [3]. The grid, which must maintain voltage frequency regulation at a certain point [4], [5], buys the electric power from the aggregator. Aggregator-based management will be important with the increase of PEVs and autonomous vehicles. More and more aggregators will get engaged in this management, which will drive them to offer competitive services, particularly to the PEVs.

An aggregator usually offer different prices for different V2G services, which are often categorized as: (i) charging, (ii) discharging, and (iii) participating in frequency regulation [6]. There can be multiple aggregators in a particular V2G system, the prices of these services can differ among them depending on various factors, specifically demand and infrastructural capacity. The aggregators compete with one another to set the optimal prices for their services. The payoff of one aggregator is greatly dependent on the prices set by the other aggregators. Therefore, one aggregator cannot independently set the prices and maximize its payoff. An aggregator also cannot charge very low prices or its business cannot be sustained. Therefore, setting the optimal prices is complicated.

In this work, we explore the problem of selecting the optimal prices using game theory, which is a strong tool used to analyze and settle down strategically conflicting issues [7]. In this game-theoretic analysis, we primarily map the competition between two aggregators using the sequential (multistage) Stackelberg game [7], [8] and solve the game to find the *Nash Equilibrium (NE)*. The Stackelberg game is extended for multiple aggregators. We extensively evaluate the results by conducting numerical simulations and analyze the results to discern the behavior of the competitors varying different factors. We also compare the sequential nature of the actions taken by the aggregator (as in the Stackelberg game) with the simultaneous nature of them by modeling the problem as a simultaneous (single stage) game – Cournot game [9].

The rest of this paper is organized as follows. Section II briefly discusses necessary background (*i.e.*, the V2G management system and pricing problem) and the challenges and objective addressed in this research. Section III discusses the pricing game model. Section IV presents the game results with

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respect to the basic two-player model, the extended multi-player model, and the Cournot model. Section V discusses the evaluation results. Section VI does a brief literature review related to this work. Section VII concludes the paper.

II. Background and Motivation

In this section, we discuss necessary background of the aggregator-based V2G management system and the corresponding pricing competition.

A. Aggregator-Based V2G Management Systems

The batteries of PEVs can consume power from the grid, provide (sell) power to the grid or participate in frequency regulation. Frequency regulation is a process of frequent charging and discharging according to the state of the power supply and load, in order to keep the frequency at a stable point. As the fluctuations of power change between positive and negative are almost evenly distributed, *i.e.*, the energy delivered and absorbed is almost equal over a long-term regulation, it is assumed that participating in frequency regulation does not change the battery's State of Charge (SOC) [10], [11].

There is a gap between the PEV's power capacity, which is 10-20 kW, and the energy provider's requirement, which is measured in MW basis [6]. Therefore, an aggregator is necessary to help organize the V2G operations and respond to the needs of many PEVs as well as the energy provider. Here, PEVs (*i.e.*, the owners of the car) subscribe to an aggregator to manage its V2G operations, while each subscriber, along with the aggregator, wants to maximize its payoff. Fig. 1 shows such an aggregator-based architecture [6], where multiple aggregators simultaneously serve PEVs with V2G services.

A PEV can subscribe to an aggregator for a time period. The aggregator has the full control to select the V2G operations for the PEV at different time slots during the subscription period. At a particular time slot, a PEV can be activated for one of the V2G services. At the time of subscription, a PEV gives all necessary information, *i.e.*, the SOC of the battery, the owner preferences (*e.g.*, the SOC of the battery at the end of the subscription), etc. to the aggregator.

B. Research Challenges and Objectives

The aggregator tasks are (i) to select the prices for V2G services for the subscribed PEVs and (ii) to schedule the actions (the V2G services) of the subscribed PEVs for each time slot. We assume that each aggregator follows the same process of scheduling the subscribed PEVs, which gives an optimal solution. We also assume that this optimal solution results in the same revenue (on average) with respect to the same number of subscribed PEVs for a particular time period. If we plan before the designated time period, the selection of strategy depends on the expected number of PEVs (and their expected properties) to be subscribed in different time slots during this period. This assumption is true when the PEVs with different properties and preferences are distributed evenly in the system (*i.e.*, the proportion of the subscribed vehicles of similar properties and preferences over the total subscribed vehicles with respect to an aggregator is the same for all aggregators).

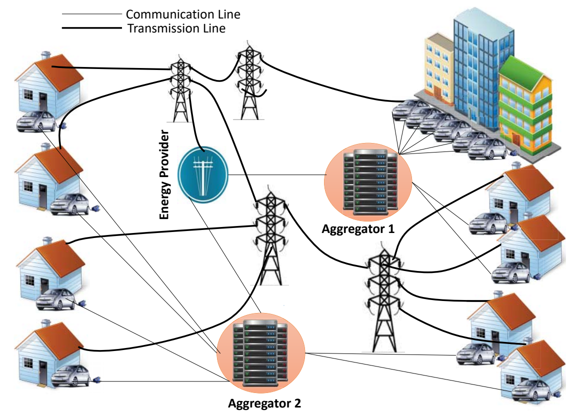

Fig. 1. An architecture of a V2G management system through aggregators.

The aggregator sets the price of charging/discharging (*e.g.*, price per kW power) and the price of participating in the frequency regulation (*e.g.*, price per kW storage capacity) for the PEVs. We assume that the game is played before the designated time period. In the game, each aggregator offers a price that can be different for different V2G services and different at the different time slots. The PEV selects an aggregator that caters to its demand accordingly. An aggregator's payoff is equal to the total revenue received from the subscribed PEVs minus the total cost paid to the grid. Total revenue received from the PEVs is a function of the price of electricity consumed for charging minus the price of discharged electricity and the price of participating in frequency regulation. If the summation of the latter two prices is higher than the first, the total received benefit is negative, *i.e.*, the aggregator needs to pay the PEVs. Similarly, total price paid to the grid is equal to the total price of consumed electricity minus the total price of discharged electricity and the price for participating in frequency regulation. However, the price received from the PEVs depends on the price rate offered to the PEVs by the aggregator, while the total price paid to the grid depends on the price rate declared by the grid. Our objective is to find the optimal prices (both the charging and discharging prices) for all the aggregators that maximize the payoff of the aggregators. We use the game-theoretic approach to find the solution. In this case, we focus on the Stackelberg game and compare it to the outcome from the Cournot game.

C. V2G Pricing Game

Pricing is a complicated issue in V2G management. We assume that price for *discharging* and price for *participating in frequency regulation* are the same and term that as *discharging price*. When an aggregator sets the prices (*charging* and *discharging* prices), other aggregators also set the prices. The aggregators compete with each other to set their optimal prices.

This competition between the aggregators can be considered as a two-player game where each of them tries to maximize its *payoff*. According to the Stackelberg game, a player known as the *leader* moves first taking the *follower*'s move into account. After this move, the second player known as the *follower* makes a move. In this paper, the terms Aggregator 1, the *leader*, and first aggregator are used interchangeably. Similarly, the terms Aggregator 2, the *follower*, and second aggregator are used interchangeably. The payoff of the aggregator during a particular period is the summation of the distinct payoffs over all the time slots during that period.

It is wroth mentioning that every aggregator needs to know the pricing information of other aggregators to optimize its offered prices and maximize the profit. The information should be as real-time as possible to achieve the best result. We assume that every aggregator is connected to a third-party server that maintains a database with the up-to-date pricing information of all the aggregators. If there is any change in prices, the server will update its database and notify the aggregators. This research addresses the "economic aspect" in V2G management through the application of game theory.

III. Formal Modeling of V2G Pricing Game

In this section, we present the V2G pricing game model. Table I shows the notations used to formalize this model.

Table I
NOTATION TABLE

Notation	Definition
P_t	Unit discharging price offered by the grid to the aggregators at time slot t
Q_t	Unit charging price offered by the grid to the aggregators at time slot t
$p_{i,t}$	Unit discharging price offered by aggregator i to its subscribers at time slot t
$p_{-i,t}$	Unit discharging price offered by aggregator(s) other than i at time slot t
$q_{i,t}$	Unit charging price offered by aggregator i to its subscribers at time slot t
$q_{-i,t}$	Unit charging price offered by aggregator(s) other than i at time slot t
$k_{i,t}$	Number of PEVs subscribed to aggregator i at time slot t
$k_{-i,t}$	Number of PEVs subscribed to aggregator(s) other than i at time slot t
N	Total number of prospective PEV customers in the market
L_i	Maximum number of PEVs served by aggregator i at a time
L_{-i}	Maximum number of PEVs served by aggregator(s) other than i at a time
$B_{j,t}$	The storage capacity of the battery of PEV j at time slot t
$T_{j,t}$	The subscription period of PEV j at time slot t
T_s	The total number of time slots
$E_{j,t}$	Total number of units consumed by PEV j at time slot t
C^S	The average cost for providing the service for a PEV by an aggregator
C_i	The fixed infrastructure cost of aggregator i

A. V2G Service Prices and Vehicle Subscriptions

A PEV may have a preference of price over time. A higher rate of charging requires a higher price. We consider this rate as a fixed one and thus the price depends on the charged amount. We model the time sensitivity with respect to the waiting time for the service. This waiting time is assumed to be dependent solely on the infrastructure capability of an aggregator that limits the number of PEVs to be served at a time. Let $p_{i,t}$ and $q_{i,t}$ be the charging and discharging prices, respectively, offered by aggregator i to PEVs at time slot t.

PEV owners choose aggregators based on their preferences. This preference generally depends on two factors: price-sensitivity and time-sensitivity. The PEV owners with a higher price-sensitivity look for aggregators that charge comparatively low prices for charging (*i.e.*, low q_i) and/or offer high prices for discharging (*i.e.*, high p_i). Therefore, the number of vehicles subscribed increases with the decrease of $q_{i,t}$ and/or the increase of $p_{i,t}$. If we consider that the PEVs are evenly interested in charging and discharging, there is a positive relationship between $p_{i,t} - q_{i,t}$ and the number of subscribed vehicles. That is, the larger is the difference, the higher the number of subscribers.

The time-sensitive PEV owners subscribe to those aggregators who can serve them with minimum waiting. There is no delay in the case for an aggregator as long as the number of vehicles subscribed to it is less than the capacity of its V2G infrastructure. Let $k_{i,t}$ be the number of vehicles subscribed to aggregator i at time t and L_i be its infrastructure capacity, which is a fixed value denoting the number of vehicles to be served at a time. When $k_{i,t}$ becomes larger than L_i, it negatively impacts the selection of aggregator i for subscription. We assume that there is no waiting time for the PEVs provided that the total infrastructure capacity of all the aggregators is more than the number of PEVs to be subscribed.

According to the above discussion, we assume the number of PEVs subscribed to aggregator i (*i.e.*, $k_{i,t}$) as follows:

$$k_{i,t} = N \times \frac{p_{i,t} - q_{i,t}}{p_{i,t} + p_{-i,t} - (q_{i,t} + q_{-i,t})} \tag{1}$$

where the following constraint should hold true:

$$k_i \leq L_i \tag{2}$$

Equation (1) calculates $k_{i,t}$ for aggregator i at time t, which is total number of PEVs subscribed to aggregator i at time t. The equation calculates $k_{i,t}$ by multiplying N with a ratio that represents the difference between the discharging price ($p_{i,t}$) and charging price ($q_{i,t}$) with the difference between the charging price of all the aggregators (*i.e.* $p_{i,t}$ and $p_{-i,t}$) and the discharging price of all the aggregators (*i.e.* $q_{i,t}$ and $q_{-i,t}$). While making the pricing decision, aggregator i must set the discharging price at least equal to the charging price; otherwise, PEV owners will not be interested in discharging. This constraint is defined as $q_{i,t} \leq p_{i,t}$. Also, the number of subscribed vehicles ($k_{i,t}$) cannot be greater than the total number of PEVs (N) in the market, i.e., $k_{i,t} \leq N$

B. Cost and Benefit

An aggregator must pay the maintenance cost with regard to the infrastructure and the variable cost related to providing services to the subscribed vehicles. Let C_i be the maintenance cost which is fixed for aggregator i and C^S be the service providing cost for each PEV. Then, the total cost G_i can be computed as follows:

$$G_i = C_i + \sum_{t=1}^{T_s} k_{i,t} C^S \tag{3}$$

The profit depends on the subscription period and the storage capacity of the battery of each subscribed PEV. Let T_j and B_j be the subscription period and the battery capacity of PEV j, respectively. Note that T_j stands for the length of the subscription period, irrespective of the time slot. It is also worth mentioning that this modeling considers time varying prices to tackle the time varying price changes. The aggregator is paid by PEV j based on the electricity it consumes. Remember that we consider that the prices for discharging and participating in frequency regulation are the same and refer to this as the discharging price. Let the aggregators sell the electricity to the power grid at the unit price of P_t and buy electricity from the grid at the unit price of Q_t. Then, the profit of an aggregator depends on the number of subscribed vehicles, the charging and discharging prices it sets, the energy consumption by each vehicle, and the time spent by each vehicle for discharging/participating in frequency regulation over the total number of time slots T_s. Let the amount of electricity PEV j consumes at time t be $E_{j,t}$.We can compute the total revenue or benefit H_i for aggregator i as follows:

$$H_i = \sum_{t=1}^{T_s} \left((q_{i,t} - Q_t) \sum_{j=1}^{k_{i,t}} E_{j,t} + (P_t - p_{i,t}) \sum_{j=1}^{k_{i,t}} T_{j,t} B_{j,t} \right) \tag{4}$$

To continue the business, each aggregator must operate at least at the break-even point (BEP) at which the total cost is equal to the total revenue [7]:

$$H_i \geq G_i \tag{5}$$

C. Payoff

Let $U_{i,t}$ be the payoff of aggregator i at time slot t. Then, the overall payoff is represented by the following equation:

$$\sum_{t=1}^{T_s} U_{i,t} = H_i - G_i \tag{6}$$

For a particular time slot t, Equation 6 is rewritten as follows:

$$\begin{aligned} U_{i,t} = (q_{i,t} - Q_t) \sum_{j=1}^{k_i,t} E_{j,t} \, + \\ (P_t - p_{i,t}) \sum_{j=1}^{k_{i,t}} T_{j,t} B_{j,t} - (C_i + k_{i,t} C^S) \end{aligned} \tag{7}$$

If we assume that each PEV charges on an average for E_t units and participates in frequency regulation (at the discharging price) for on an average T time with each PEV having the battery capacity of B_t units at time slot t, we can rewrite Equation (7) in the following way:

$$\begin{aligned} U_{i,t} &= (q_{i,t} - Q_t) k_{i,t} E_t + \\ &\quad (P_t - p_{i,t}) k_{i,t} T \times B_t - C_i - k_{i,t} C^S \\ &= k_{i,t}((q_{i,t} - Q_t) E_t + \\ &\quad (P_t - p_{i,t}) T \times B_t - C^S)) - C_i \end{aligned} \tag{8}$$

Now, taking the value of $k_{i,t}$ from Equation (1), Equation (8) becomes the following:

$$\begin{aligned} U_{i,t} =& N \times \frac{p_{i,t} - q_{i,t}}{(p_{i,t} + p_{-i,t}) - (q_{i,t} + q_{-i,t})} \times \\ & ((q_{i,t} - Q_t) E_t + (P_t - p_{i,t}) T \times B_t - C^S) - C_i \end{aligned} \tag{9}$$

The following section presents the solution of this game model using Equation (9).

IV. Solutions of The Pricing Games

We first solve the game based on the Stackelberg game for two aggregators. Later, we extend the game model for multiple aggregators. We do not take into account any privacy issue regarding the PEV driver's information in solving the game.

A. Two-Aggregator Stackelberg Game

In the case of the Stackelberg game, we consider Aggregator 1 as the *leader* and Aggregator 2 as the *follower*. Aggregator 2 plays the best response to Aggregator 1's action. Aggregator 2 sets its charging price (q_2) in such a way that maximizes its payoff given the action of the Aggregator 1. Once Aggregator 2 sets its optimum charging price, it sets the discharging price (p_2). When Aggregator 1 sees the (new) prices set by Aggregator 2, it will set its optimum charging price (q_1) and consequently the optimum discharging price (p_1). This process is played back and forth several times until both of them reach to *Subgame Perfect nash Equilibrium* (SPNE) [7] (*i.e.*, the convergence point). Fig. 2 is a simplified diagram of the proposed process corresponding to Stackelberg game. In the following, we discuss the computations that are executed during this process at a particular time

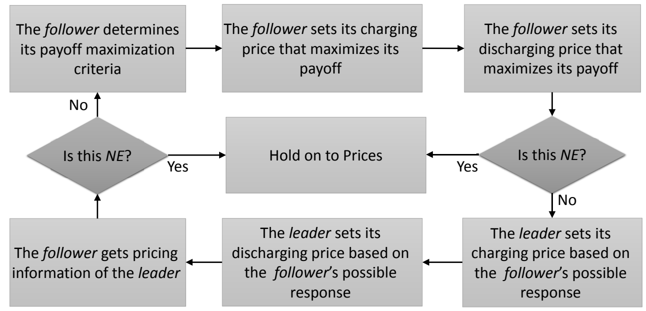

Fig. 2. Process flow of Stackelberg game. Here, Aggregator 1 works as the *leader* and Aggregator 2 as the *follower.*

slot t. According to Equation (9), the payoff of Aggregator 2 (U_2) is written as:

$$
\begin{aligned}
U_{2,t} =& N \times \frac{p_{2,t} - q_{2,t}}{p_{1,t} + p_{2,t} - (q_{1,t} + q_{2,t})} \times \\
& ((q_{2,t} - Q_t)E_t + (P_t - p_{2,t})T \times B_t - C^S) - C_2
\end{aligned} \tag{10}
$$

Similarly, we reach the payoff of Aggregator 1 (U_1).

In this sequential game, Aggregator 2 has *perfect* and *complete* information about Aggregator 1's pricing decision. At the first step, we need to find the best response of Aggregator 2 based on the actions of Aggregator 1. Taking the first order derivative of $U_{2,t}$ with respect to $q_{2,t}$ and considering $\frac{\partial U_{2,t}}{\partial q_{2,t}} = 0$ when $U_{2,t}$ is maximum, we get:

$$
\begin{aligned}
0 =& (N \times \frac{p_{2,t} - q_{2,t}}{p_{1,t} + p_{2,t} - q_{1,t} - q_{2,t}} \times E_t + \\
& ((q_{2,t} - Q_t)E_t + (P_t - p_{2,t})T \times B_t - C^S) \times N \times \\
& \frac{q_{1,t} - p_{1,t}}{(p_{1,t} + p_{2,t} - q_{1,t} - q_{2,t})^2}
\end{aligned} \tag{11}
$$

Equation (11) provides us the *follower*'s best response $q_{2,t}(q_{1,t})$, the optimal value for $q_{1,t}$. Using this optimal $q_{2,t}$, we find the *follower*'s best response $p_{2,t}(p_{1,t})$. Let $q^*_{2,t} = q_{2,t}(q_{1,t})$ and plugging this value to Equation (10), we get:

$$
\begin{aligned}
U_{2,t} =& (N \times \frac{p_{2,t} - q^*_{2,t}}{p_{1,t} + p_{2,t} - q_{1,t} - q^*_{2,t}} \times \\
& ((q^*_{2,t} - Q_t)E_t + (P_t - p_{2,t})T \times B_t - C^S) - C_2
\end{aligned} \tag{12}
$$

Performing the first order partial differentiation on Equation (12) with respect to p_2, we have:

$$
\begin{aligned}
0 =& (N \times \frac{p_{2,t} - q^*_{2,t}}{p_{1,t} + p_{2,t} - q_{1,t} - q^*_{2,t}} \times (-T \times B_t) + \\
& ((q^*_{2,t} - Q_t)E_t + (P_t - p_{2,t})T \times B_t - C^S) \times \\
& N \times \frac{p_{1,t} - q_{1,t}}{(p_{1,t} + p_{2,t} - q_{1,t} - q^*_{2,t})^2}
\end{aligned} \tag{13}
$$

Let the solution of the above equation be $p^*_{2,t}$ where $p^*_{2,t} = p_{2,t}(p_{1,t})$. We now need to find the *leader*'s response given the *follower*'s actions. Plugging $p_{2,t} = p^*_{2,t}$ and $q_{2,t} = q^*_{2,t}$ in the payoff of Aggregator 1 ($U_{1,t}$) and taking its first order partial derivative with respect to $q_{1,t}$, we can easily get Aggregator 1's best move for $q_{1,t}$, *i.e.*, the optimal value of $q_{1,t}$; let it be $q^*_{1,t}$. Now, $p^*_{1,t}$, the best move of Aggregator 1 for $p_{1,t}$, can be found by plugging Aggregator 2's best responses (*i.e.*, $q^*_{2,t}$ and $p^*_{2,t}$) and Aggregator 1's best move for $q_{1,t}$ (*i.e.*, $q^*_{1,t}$) to the payoff equation of the Aggregator 1 and solving the equation by taking first order derivative with respect to $p_{1,t}$.

B. Multi-Aggregator Stackelberg Game

We extend the Stackelberg game to include n aggregators. The schematic game model is shown in Fig. 3. The model shows that one aggregator is randomly chosen out of n aggregators as the *leader* and the rest $n-1$ aggregators work as the *follower* group. Once the leader and the follower have been selected, the leader looks to optimize its total profit over

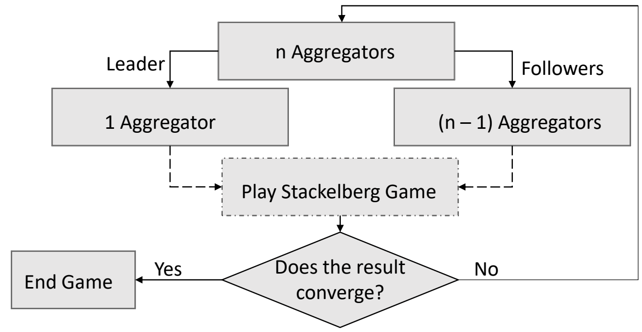

Fig. 3. Stackelberg game for Multiple Aggregators.

the time slots (for our experiment we consider 8 time slots) and the Stackelberg game is played. For example, if the aggregator i is the leader, the rest of the aggregators (denoted as $-i$) work as the follower group and aggregator i finds optimum values of charging ($q^*_{i,t}$) and discharging ($p^*_{i,t}$) prices according to the procedure described in Section IV-A. These optimum prices maximize its payoff function described in Equation (9). In this case, ($q_{-i,t}$) and discharging ($p_{-i,t}$) represent the average charging and discharging prices, respectively, of other aggregators ($-i$).

The game result is checked for convergence against a threshold value. The game is said to be converged if the result does not change for a good number of times. In our case, the convergence criterion is met if the game result does not change for n number of times because in this way, all the aggregators will have played the game without changes in their result. However, if the game does not converge, the game result is updated and becomes available to other aggregators. The follower aggregators observe this result. In the next step, the leader aggregator from the previous iteration merges with the $n-1$ follower aggregators. The next leader is chosen from the $n-1$ follower aggregators of the previous iteration. The game continues until convergence is met. At the end of the game, every aggregator will have its optimized charging and discharging price for each of the time slots. However, if any aggregator deviates from these optimal prices that particular aggregator will be economically disadvantaged and other aggregators will not be negatively affected by the deviated aggregator's behavior.

C. Cournot Game

The Cournot game is defined as a single-shot game [12]. In this type, both the players have the complete but imperfect information about each other. In a Cournot game, equilibrium is reached when each firm correctly assumes the opponents' output and chooses a level of output that maximize its own profits. There is no incentive for either firm to change from this equilibrium. When this game is played, both the aggregators are unsure about their opponent's strategy. That is, Aggregator 1 takes action without knowing the move made by Aggregator 2. Although possible actions of each other are known

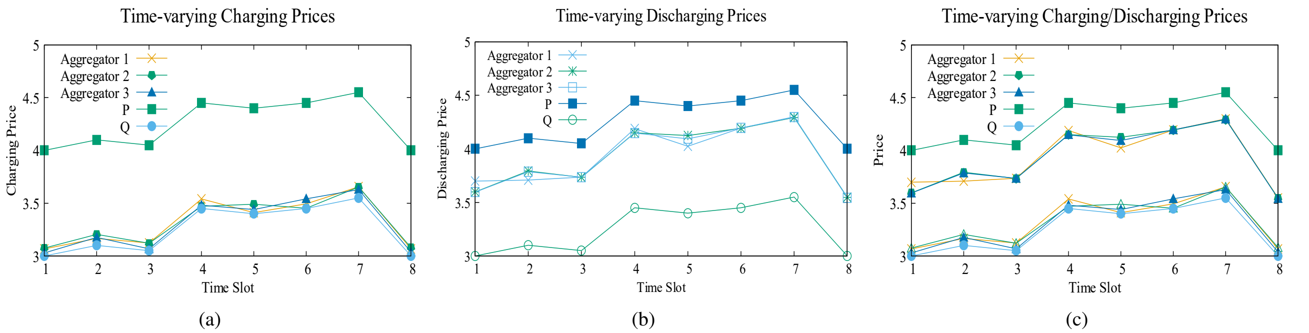

Fig. 4. Optimal prices (*i.e.*, at the equilibrium of the Stackelberg game) for different aggregators in different time slots: (a) charging prices, (b) discharging prices, and (c) charging and discharging prices.

previously, the actual strategy, which is played in real-time, is unknown to each other. It is to be noted that a strategy is a set of possible actions. Before taking action, each aggregator comes up with its best move that results in maximizing payoff. As both of them act simultaneously, they need to set their action profile beforehand.

The cost and benefit follow the same procedures described in Section III-B. In this case, each aggregator takes step simultaneously to maximize its payoff defined in Equation (9). In our paper, the aggregators first choose charging prices (i.e., q_i) simultaneously in such a way that maximize their payoffs. After that, the aggregators set discharging prices (i.e., q_i) that maximize their payoff. The game is played for each time slot and optimal values for charging and discharging prices are chosen in such a way to maximize the total payoff over all the time slots for each aggregator.

V. Evaluation

We extensively analyze the game by conducting numerical simulations on synthetic V2G scenarios, and find the equilibriums. In the following, we present the evaluation methodology and the results of the games with brief discussions.

A. Methodology

We implement the solution of the game in C/C++ using IBM ILOG CPLEX Optimization Studio tool. We consider three aggregators and divide 24 hours into eight time slots with each time slot having three hours. We present the analysis on the Stackelberg game and the Cournot game, respectively. We arbitrarily choose the values for the system parameters, including P_t and Q_t. It is worth mentioning that, these values are purely experimental and do not represent the actual values in the market. These arbitrary values may impact the results of the game, but not its behavior.

Tables II and III show the values that we consider for different game parameters. It is to be noted that the changes in arbitrary values (*e.g.*, Ps and Qs) will only impact the numerical values of the results but it will not impact how one aggregator will react to other aggregators' price changes. More specifically, different P and Q values at the various time slots will change the offered price values but not the way an aggregator behaves to other aggregator's price changes. Although the proposed game model can be applied for any number of aggregators, we consider three aggregators to keep our discussion tractable.

B. Characteristics Analysis of Stackelberg Game

The target of each aggregator is to maximize its payoff. Each of the aggregators plays for eight time slots and finds prices for respective time slots. Fig. 4(a) shows the charging prices of the aggregators at these time slots, including the corresponding charging and discharging prices of the grid. We observe that each aggregator keeps its charging price very close to the grid's charging price (Q_t) at every time slot. By setting a charging price close to the grid's price, an aggregator is making revenues mostly to meet the expenditure of paying the grid for the aggregated charging of the PEVs subscribed to it. However, we see a profit-incurring scenario in the case of discharging service. Fig. 4(b) shows the discharging prices of the aggregators at those time slots and we can see that the discharging prices are significantly lower than the grid's discharging price (P_t). As a result, the aggregator is paying less to the PEV owners for discharging than the price it receives from the grid for the aggregated discharging.

Fig. 4(c) presents both charging and discharging prices of the aggregators, and we can see that discharging prices are significantly high compared to the charging prices; however, the grid's discharging price is much higher than its charging price. As we just have seen, an aggregator is making profit by setting its discharging price (at which it pays money to the participant) lower than the grid's discharging price (at which it receives money from the grid). However, its charging price (at which the vehicles pay it) is mostly just above the the grid's charging price, which mostly equalize the revenue with the cost. It can easily comprehend that the smaller the gap between the charging and discharging prices of an aggregator, which are must be within the spread of the grid's charging and discharging prices, the larger is the profit. However, the discharging price must be greater than the charging price to attract the PEV owners to participate in discharging.

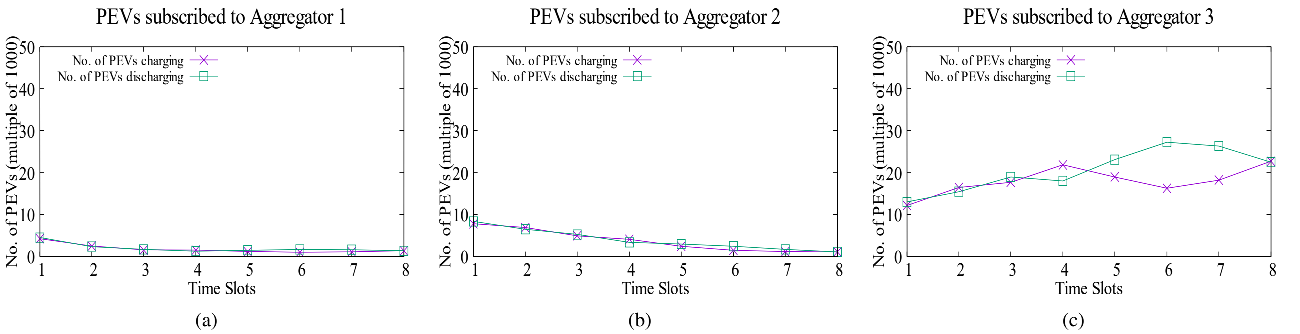

Fig. 5. At the equilibrium of the Stackelberg game, the number of subscribed PEVs to different aggregators at different time-slots of the day: (a) Aggregator 1, (b) Aggregator 2, and (c) Aggregator 3.

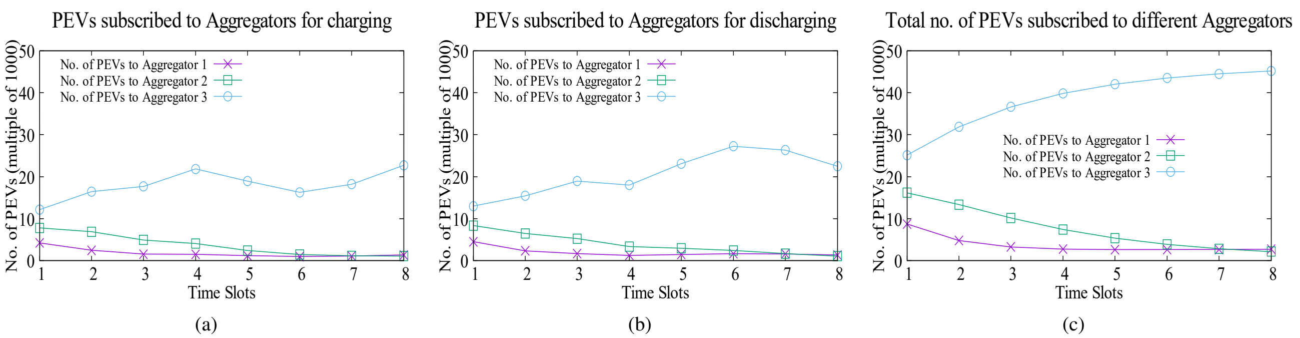

Fig. 6. At the equilibrium of the Stackelberg game, (a) the number of PEVs subscribed for charging, (b) the number of PEVs subscribed for discharging for different aggregators, and (c) the total number of PEVs subscribed to each aggregators.

Table II
PARAMETER VALUES USED IN SIMULATIONS

Time Slot	1	2	3	4	5	6	7	8
P_t	4	4.1	4.05	4.45	4.4	4.45	4.55	4
Q_t	3	3.1	3.05	3.45	3.4	3.45	3.55	3

The grid can vary their selling (P_t, corresponding to charging) price and buying (Q_t, corresponding to discharging) price in every time slot. As shown in Fig. 4(a) and Fig. 4(b), all the aggregators change their pricing to accommodate the grid price changes. Every aggregator maintains a spread between the charging price and the discharging price. We observe that, compared to the other two aggregators, Aggregator 1 offers a relatively lower charging price (Fig. 4(a)) and relatively higher discharging price (Fig. 4(b)), which is an attractive offering for the PEVs. This is because, the infrastructure capacity of Aggregator 1 is less than that of the other two aggregators (see Table III), which might deter the PEVs going to Aggregator 1 because of its limited capacity. The aggregator compensates this shortfall by offering attractive prices.

C. Performance Analysis of Stackelberg Game

Every aggregator has different infrastructure capacity. The total number of subscription to each aggregator might be influenced by this capacity. We assume that Aggregator 1, Aggregator 2, and Aggregator 3 have three different capacities, 100000, 200000, and 300000. The resulting subscription numbers for each of the aggregators are shown in Fig. 5(a), 5(b), and 5(c), respectively. The number of PEVs subscribed to different aggregators varies in different time slots. Fig. 6(a) and 6(b) show the number of vehicles subscribed for charging and discharging, respectively to different aggregators. We observe that Aggregator 1 and Aggregator 2 maintain close subscription number both for charging and discharging while the subscription number differs in the case of Aggregator 3. This variation lies in the fact that every aggregator plays for every time slot and tries to maximize their aggregated payoffs. Fig. 6(c) shows total number of PEVs subscribed to each of the three aggregators at different time slots. The figure shows that Aggregator 3 has the highest number of subscribed PEVs of the three aggregators at every time slot. The reason behind this lies in the fact that the infrastructure capacity plays the dominant role in determining the number of subscribed vehicles as in this case Aggregator 3 has a higher infrastructure capacity than that of the other two aggregators.

D. Profit Analysis of Stackelberg Game

The purpose of all the aggregators is to increase their profits, where maximizing profit necessarily means maximizing pay-

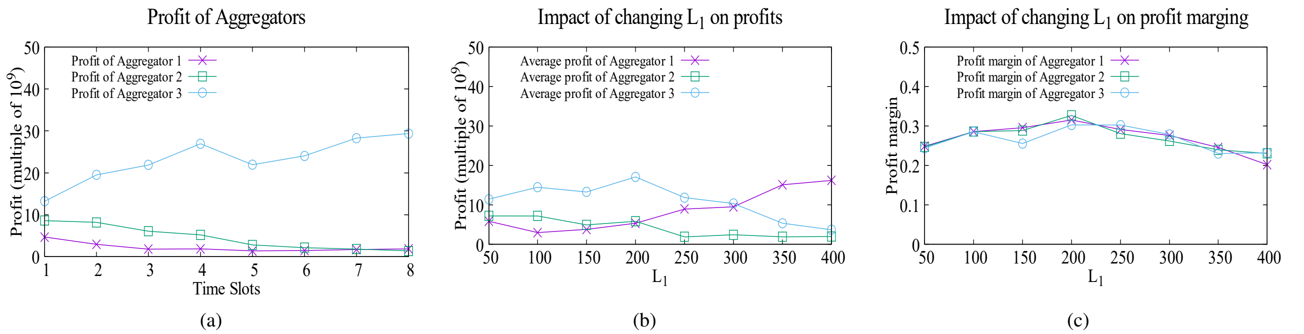

Fig. 7. At the equilibrium of the Stackelberg game, (a) profits of different aggregators with infrastructure capacity (L_i) fixed, (b) profits of different aggregators with changing L_1 , and (c) profit margin of different aggregators with changing L_1.

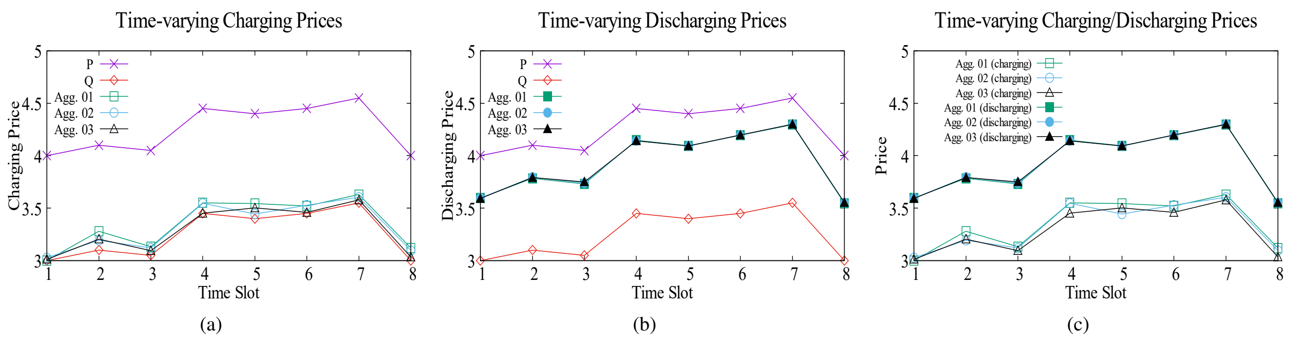

Fig. 8. At the Cournot game model's equilibrium each aggregator selects its optimal prices at different time-slots: (a) charging prices, (b) discharging prices, and (c) both charging and discharging prices.

Table III
PARAMETER VALUES USED IN SIMULATIONS (MULTIPLE OF 1000)

N	L_1	L_2	L_3	C_1	C_2	C_3	C^S
1000	100	200	300	30	30	30	0.2

off. We observe in Fig. 7(a) that the profit of Aggregator 3 is higher than that of the other two aggregators. This is because the number of PEVs subscribed to the aggregator is also higher than the other two aggregators (see Fig. 6(c)), and the number of PEVs subscribed to a particular aggregator positively reflects in profit of that aggregator. The increase in infrastructure capacity means the aggregator can serve more PEVs, which ultimately results in increased subscribed vehicles (see Equation (2) in Section III). As a result, profit increases with the increase of infrastructure capacity. We observe this scenario in Fig. 7(b), where the profit of Aggregator 1 increases with the increase of its infrastructure capacity (L_1).

However, increase in infrastructure capacity might not always be a good thing for aggregators because increase in infrastructure capacity also increases cost. When cost increases, it impacts profit margin (PM), which is the ratio of net profit and revenue. Fig. 7(c) shows how profit margins of different aggregators react with the change in infrastructure capacity of Aggregator 1 (L_1). We observe that PM of every aggregator is impacted by the change in L_1. When L_1 increases, the number of subscribed PEVs also increases, which somewhat impacts the number of subscribed PEVs to other aggregators. We observe in Fig. 7(c) that the PM of Aggregator 1 increases at some point and then it decreases. The PM increases when the number of subscribed PEVs nears the infrastructure capacity and it decreases when the PM becomes far less than the infrastructure capacity.

E. Characteristics Analysis of Cournot Game

In the Cournot game experiments, the target of each aggregator remains the same as the Stackelberg Game – maximizing its payoff. The charging prices, according to Cournot game's results, are shown in Fig. 8(a). We observe that each aggregator keeps its charging prices close to the grid's charging price (Q_t). We also observe that, the charging prices chosen by Aggregator 1 and Aggregator 2 are almost the same; however, the price chosen by Aggregator 3 shows a slightly higher price compared to the other two. This is because both Aggregator 1 and Aggregator 2 have almost the same infrastructure capacity, whereas the capacity of Aggregator 3 is much larger. While a smaller charging infrastructure can become full soon, a larger capacity can hold more vehicles, thus creating a potential to receive a higher profit.

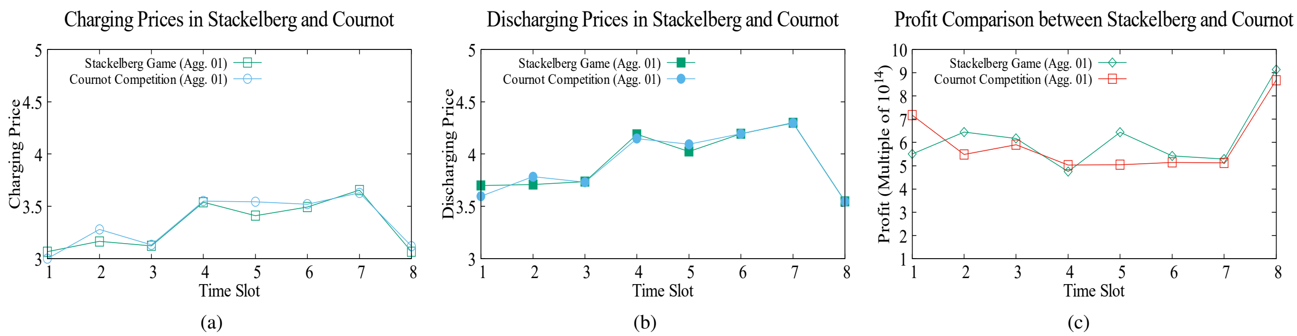

Fig. 9. Comparison between the Cournot game and the Stackelberg game with respect to (a) charging price, (b) discharging price, and (c) profit at their respective equilibriums.

Table IV
PERCENTAGE INCREASE IN PROFITS OF AGGREGATORS IN STACKELBERG GAME COMPARED TO COURNOT GAME

Time Slot	1	2	3	4	5	6	7	8
Aggregator 1	-23.4	17.6	4.5	-5.6	27.7	5.5	3.2	5.3
Aggregator 2	-3.5	0.8	2.9	3.7	13.3	-2.2	8.8	5.3
Aggregator 3	-9.8	-13.6	12.6	16.6	0.8	0.4	7.7	12.1

We observe a contrasting scenario in Fig. 8(b), where discharging prices of the aggregators are significantly low from the grid's discharging price (P_t). We observe the charging and discharging prices together in Fig. 8(c). The gap between the charging price and the grid's charging price is smaller than that of the discharging price and the grid's discharging price. This behavior shows that, similar to the Stackelberg game, aggregators are more willing to make profit from the discharging price than the charging price. The rationale behind this lies in the fact that discharging price is always kept higher than the charging price so that aggregators can attract customers to participate in the discharging process.

F. Comparison of Stackelberg with Cournot Game

In the Cournot game, players are unsure about each others action, *i.e.*, each of them plays the game with imperfect information. In this case, both the aggregators play simultaneously and unlike in the Stackelberg game, there is no *leader* or *follower* concept. Here, we discuss the change in pricing behavior because of the game selection for Aggregator 1.

The optimal charging prices in both of the games are shown for each of the time slots in Fig. 9(a). We observe that, except for the first time slot, the charging price in the case of the Stackelberg game is never less than that of the Cournot game. However, the discharging price does not follow any such pattern as we observe in Fig. 9(b). In this case, the discharging price of the Stackelberg game sometimes goes below and sometimes goes above that of the Cournot game.

However, we observe in Fig. 9(c) that, except for time slots 1 and 2, the profit in the case of the Stackelberg game is always higher than that of the Cournot game. Table IV shows the percentage increase (decrease) in profits in the case of the Stackelberg game compared to the Cournot game for different aggregators in different time slots. We find that in the case of the Stackelberg game, Aggregators 1, 2, and 3 experience a profit increase of 3.30%, 3.34%, and 3.05%, respectively, compared to the Cournot game. The reason behind this is in the case of the Stackelberg game, the aggregators can observe the result of other aggregators and take actions accordingly to maximize their profit, whereas, in the case of the Cournot game, the aggregators do not have any such option. It is worth mentioning that, although 3% of profit increase does not sound high as a number, we are considering a market where there will be innumerable PEVs in future. In this market, aggregators can make profits in millions and a 3% increase will have a huge impact on the profitability.

VI. RELATED WORK

There have been varieties of V2G related works in recent years. Mets *et al.* assessed the optimal car battery (dis)charging scheduling to achieve peak saving and reduction of the variability (over time) [13]. Shi *et al.* studied the real-time V2G control problem under price uncertainty and modeled the electricity price as a Markov chain with unknown transition probabilities [14]. Wu *et al.* proposed an operating framework for aggregators of plug-in electric vehicles [15]. Kempton *et al.* studied the rationale and economic motivation for social advantages of V2G systems [16]. Guille *et al.* assessed the deployment of an aggregation of battery vehicles for the provision of frequency regulation requiring fast response time and energy supply for peak saving [4]. Hutson *et al.* proposed an intelligent method for scheduling usage of available energy storage capacity from plug-in hybrid electric vehicles (PHEV) and electric vehicles (EV) [17]. Yilmaz *et al.* discussed the impact of vehicle-to-grid technologies on distribution systems and utility interfaces [18].

Kintner *et al.* and Scott *et al.* investigated the technical potential [19] and economic impacts [20] respectively, of using the existing idle capacity of the electric infrastructure in conjunction with the emerging PHEV technology to meet the majority of the daily energy needs of the U.S. LDV fleet.

Bradley *et al.* presented the basic design considerations for PHEVs, including vehicle architecture, energy management systems, drive-train component function, energy storage trade-offs and grid connections [21]. Han *et al.* conducted the economic feasibility analysis of V2G frequency regulation taking the battery life in consideration [22]. Tomic *et al.* evaluated the economic potential of two utility-owned fleets, equipped with battery-electric vehicles [5]. A smart charging strategy has been discussed in [23]. An agent-based approach was used and a control software was developed for charging processes of an electric vehicle fleet in a smart grid architecture in [24]. In [6], an optimal scheduling framework for participating in V2G services by the PEVs, controlled by an aggregator, was proposed.Wang *et al.* discussed on how EVs could participate in frequency regulation and they proposed a cloud-based service where every EV in an area is assembled into a cluster agent to harmonize their frequency response [25]. In [26], the authors, discussed how the estimated capacities from the V2G system can be used for establishing a regulation contract. Maharjan *et al.* proposed a Stackelberg game model for the interaction between operators [27]. Zeng *et al.* proposed an auction-based group-selling approach for Demand Response Management in V2G Systems [28].

None of the above discusses the interaction among the aggregators in setting their prices. In this work, we address this issue using a game-theoretic approach, which is unique to the best of our knowledge. It is worth mentioning that the works by Maharjan *et al.* [27] and Zeng *et al.* [28] are closest to our work. However, as we just have discussed, the first work deals with the demand response service, while the second work form an optimal group of mulitple PEVs to sell prices to the grid. On the contrary, in our work aggregators are active players, while PEVs play passive role. Finally, we also provided competition models between a fuel provider and an electric provider in [29].

VII. Conclusion

Game theory has been proven to be an important tool in making strategic decisions in complex situations. The application of game theory is suited perfectly to resolving the intricate pricing competition issue among aggregators. The V2G technology is still in an evolving stage and the aggregators are important part of it. We have applied Stackelberg Leadership Model to capture the competition between the aggregators. The solution to the corresponding game provides optimal prices for every aggregators for every time slot. Since all the optimal prices are chosen based on the Nash equilibrium, no aggregator can gain more profit by deviating from the equilibrium point. We evaluate the game results on synthetic problems and analyze how different factors impact the decision. We also showed that the Stackelberg game is better suited in the V2G market scenario than the Cournot Competition.

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